ELECTROMAGNETIC WAVES — JEE MAIN PHYSICS

Comprehensive Formula Sheet for Revision

1. DISPLACEMENT CURRENT — FULL TREATMENT

$$i_d = \varepsilon_0 \frac{d\Phi_E}{dt}, \qquad J_d = \varepsilon_0 \frac{\partial E}{\partial t} \tag{1}$$

Charging capacitor (plate area A, separation d):

$$i_d = C\frac{dV}{dt} = \varepsilon_0 A \frac{dE}{dt} \tag{2}$$

Magnetic field due to displacement current (plate radius R):

$$B = \frac{\mu_0 i_d\, r}{2\pi R^2} \ (r \le R), \qquad B = \frac{\mu_0 i_d}{2\pi r} \ (r \ge R) \tag{3}$$

2. MAXWELL'S EQUATIONS (Differential Form)

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \qquad \text{(Gauss — E)} \tag{4}$$

$$\nabla \cdot \vec{B} = 0 \qquad \text{(Gauss — B)} \tag{5}$$

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \qquad \text{(Faraday)} \tag{6}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0\varepsilon_0 \frac{\partial \vec{E}}{\partial t} \qquad \text{(Ampere–Maxwell)} \tag{7}$$

3. WAVE EQUATION & SPEED (Derivation Result)

Combining Faraday's and Ampere–Maxwell's laws in free space ($\rho = 0$, $J = 0$) gives the wave equations:

$$\frac{\partial^2 E}{\partial x^2} = \mu_0\varepsilon_0 \frac{\partial^2 E}{\partial t^2}, \qquad \frac{\partial^2 B}{\partial x^2} = \mu_0\varepsilon_0 \frac{\partial^2 B}{\partial t^2} \tag{8}$$

$$c = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = 3\times 10^8 \text{ m/s}, \qquad n = \frac{c}{v} = \sqrt{\varepsilon_r \mu_r} \tag{9}$$

- Plane wave form: $E = E_0 \sin(kx - \omega t)$, $B = B_0 \sin(kx - \omega t)$
- E and B in phase; $\frac{E_0}{B_0} = \frac{\omega}{k} = c$
- $k = \frac{2\pi}{\lambda}$, $\omega = 2\pi f$

4. POYNTING VECTOR (Vector Form)

$$\vec{S} = \frac{1}{\mu_0}(\vec{E}\times\vec{B}) \tag{10}$$

- Instantaneous magnitude: $S = \frac{EB}{\mu_0} = c\varepsilon_0 E^2$
- Time-averaged (Intensity):

$$I = \langle S \rangle = \frac{1}{2}\varepsilon_0 c E_0^2 = \frac{cB_0^2}{2\mu_0} = \frac{E_0 B_0}{2\mu_0} \tag{11}$$

- Point source at distance r: $I = \frac{P}{4\pi r^2}$
- RMS relation: $E_{rms} = \frac{E_0}{\sqrt{2}}$, so $I = \varepsilon_0 c E_{rms}^2$

5. ENERGY DENSITY — PROOF OF EQUAL SHARING

$$u_E = \frac{1}{2}\varepsilon_0 E^2, \qquad u_B = \frac{B^2}{2\mu_0} \tag{12}$$

$$\text{Using } E = cB \text{ and } c^2 = \frac{1}{\mu_0\varepsilon_0} \Rightarrow u_E = u_B \tag{13}$$

$$u_{total} = \varepsilon_0 E^2 = \frac{B^2}{\mu_0}, \qquad u_{avg} = \frac{1}{2}\varepsilon_0 E_0^2 \tag{14}$$

6. MOMENTUM, RADIATION PRESSURE & FORCE

$$p = \frac{U}{c}, \qquad \text{momentum density } g = \frac{S}{c^2} = \frac{u}{c} \tag{15}$$

- Fully absorbing: $P_{rad} = \frac{I}{c}$
- Fully reflecting (normal incidence): $P_{rad} = \frac{2I}{c}$
- Partially reflecting (reflectance R): $P_{rad} = \frac{I}{c}(1+R)$
- Force on surface: $F = P_{rad} \times A$
- Oblique incidence at angle θ: effective area $= A\cos\theta$

7. LC OSCILLATOR & GENERATION

$$f = \frac{1}{2\pi\sqrt{LC}} \tag{16}$$

- Accelerated charges radiate EM waves at their oscillation frequency
- Practical LC oscillators generate radio-frequency waves only (limited by circuit losses at high f)

8. ELECTROMAGNETIC SPECTRUM — PRODUCTION & DETECTION

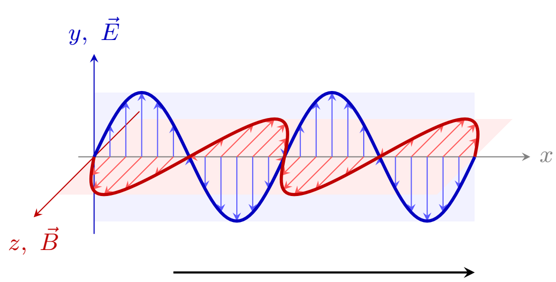
- **Radio** (> 0.1 m): LC circuits $\rightarrow$ detected by antenna/receiver
- **Microwave** (0.1 m–1 mm): Klystron, magnetron $\rightarrow$ point-contact diodes

- **Infrared** (1 mm−700 nm): Hot bodies, molecular vibration → thermopiles, bolometers
- **Visible** (700 − 400 nm): Electron transitions in atoms → eye, photocells
- **UV** (400 nm−1 nm): Sun, mercury vapour lamps → photocells, photographic film
- **X-rays** (1 nm−10^{-3} nm): Electron bombardment of metal targets → photographic plates, GM counters
- **Gamma** ($< 10^{-3}$ nm): Nuclear decay/de-excitation → GM counters, scintillation detectors

9. HIGH-YIELD NUMERICAL RELATIONS

- Solar constant relation: $I = \dfrac{P_{sun}}{4\pi r^2}$ (r = Earth–Sun distance)
- Given average power P of an isotropic source, field amplitude at distance r: $E_0 = \sqrt{\dfrac{\mu_0 c P}{2\pi r^2}}$
- Peak values from average intensity: $E_0 = \sqrt{2I/(\varepsilon_0 c)}$, $B_0 = E_0/c$
- Dimensional check: $[\varepsilon_0 E^2] = [B^2/\mu_0]$ = energy density (J/m^3)

KEY DIAGRAMS



Propagation ($+x$)

$$\vec{S} = \frac{1}{\mu_0}\vec{E}\times\vec{B}$$

Blue: electric field in the xy plane.
Red: magnetic field in the xz plane.
$E_y = E_0\sin(kx-\omega t)$, $B_z = B_0\sin(kx-\omega t)$
$\vec{E}\perp\vec{B}$, $\vec{E}\perp\vec{S}$, $\vec{B}\perp\vec{S}$.
Fields oscillate in phase; amplitudes use separate scales.

MOST IMPORTANT FORMULAS AT A GLANCE

Concept	Key Formula
Displacement current density	$J_d = \varepsilon_0 \dfrac{\partial E}{\partial t}$
Speed of EM wave	$c = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}$
E–B amplitude ratio	$E_0/B_0 = c$
Intensity (avg. Poynting)	$I = \dfrac{1}{2}\varepsilon_0 c E_0^2$
Momentum	$p = U/c$
Pressure (absorb/reflect)	I/c , $2I/c$
LC oscillator frequency	$f = \dfrac{1}{2\pi\sqrt{LC}}$
B field inside charging capacitor	$B = \dfrac{\mu_0 i_d r}{2\pi R^2}$

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